Employing Theatrical Games to Establish and Support Onlife Learning Communities on Mathematical Principles of Informatics

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Abstract. This paper presents how the PerFECt Framework for establishing and sustaining learning communities can be combined with innovative approaches of teaching mathematics using drama, to develop unplugged activities to introduce the binary system in primary and secondary school students. The resulting Human Calculator theatrical game enables students, using their bodies, to learn the binary representation of numbers and discover the algorithms of addition, subtraction, multiplication and division.

Keywords: Onlife Communities, Theatrical Games, Teaching Informatics, Binary System.

1 Introduction

Seymour Papert, a pioneer of educational informatics, has emphasized the need to give opportunities to children to become better learners through computer coding. In his seminal book “Mindstorms: Children, Computers and Powerful Ideas” he presented the big ideas behind computers and how these big ideas can be better understood by young children (Papert, 1980). In later works, Papert expressed his disappointment that readers of this book that subsequently tried to put its vision into action through their teaching, focused too much on the word “computers”, paying less attention to “powerful ideas”. Today, with the advent of educational robotics, STEM/STEAM
teaching and the implementation of world-wide initiatives to promote coding such as the Hour of Code and EU Week of Code (Kyfonidis et al., 2017; Moumoutzis et al., 2017a), the danger of overemphasizing the use of computers without paying the necessary attention to the powerful ideas behind them, is even bigger.

The issue becomes even more important when it comes to younger children at the first grades of primary school or even kindergarten. There is already a plethora of related apps and robotic kits available for children at these ages. Consequently, there is a temptation for teachers to focus on the technical part of educational informatics instead of trying to emphasize the technology independent big ideas of computing so that young children could deeply understand and effectively use digital technologies throughout their lives.

The work presented in this paper tries to address all these concerns on overemphasizing the technical aspects of computers in education by putting an explicit focus on powerful ideas related to the mathematical foundation of computer science and trying to offer engaging learning activities for children to understand these ideas even without using actual computer equipment. This is in line with an interesting approach to educational informatics termed “Computer Science Unplugged” (Bell & Vahrenhold, 2018). To make the unplugged activities more interesting and engaging, we employ ideas from the TIM methodology that links mathematical teaching and learning with theatre in education. The methodology is developed and promoted by the Erasmus+ project TIM – Theatre in Mathematics (https://www.theatreinmath.eu/). The resulting learning framework is titled “Human Calculator” and supports the creative exploration of binary representation of integers and the algorithms that implement addition, subtraction, multiplication and division.

The overall approach is based on PerFECT, a design framework that emphasizes performativity, i.e. that knowledge is based on human performance and actions done within certain social contexts, rather than development of conceptual representations. This framework has already been used to guide the developments on several projects addressing digital heritage preservation and creative learning (Moumoutzis et al., 2018; Moumoutzis et al., 2019). In the work reported in this paper, the PerFECT framework is re-interpreted and re-used to focus on the idea of de-design. De-design addresses the need to put emphasis on leaving out features from a design in order to give more freedom to the users of a certain system (Cabitza, 2014). In the case of the Human Calculator theatrical game, de-design is used to offer an alternative to using digital tools for the learning topic under consideration: the human body is activated and put in action to enable all the necessary interactions that engage the participating children in the learning process.

The rest of the paper is organized as follows: Section 2 presents the design principles of the PerFECT framework that are employed in the development of the Human Calculator theatrical game. Section 3 presents the core features of the Human Calculator theatrical game and links it to Mathemart, the drama-based teaching methodology employed in the TIM project. Section 4 presents how this game can be used to explore addition and multiplication. Section 5 extends the basic version of the game to enable the exploration of subtraction and division. Section 6 presents major conclusions and future plans.
Design Principles Adopted from the PerFECt Framework

Technology in general and digital technologies in specific is a catalyst for establishing and sustaining certain social structures as Cabitza et al. (2014) underline and exemplify. They emphasize the fact that end users are becoming “producers” of contents and functionalities. On the other hand, the term expert user is suggested to signify an expert in a particular domain with main goal to develop the technological capabilities available on that domain.

An expert user engages in creative/authoring activities without being a professional software developer. Usually the role of end user and that of an expert user are played by different people that may also belong to different communities. Furthermore, Cabitza & Simone (2015) suggest the role of meta-designer to describe the work done by professionals who create the socio-technical conditions for empowering end users in operating as active contributors of contents and functionalities.

A meta-designer creates open systems that can be further developed by their users acting as co-designers. However, apart from the technical conditions necessary to set up such environments, there is a need to effectively create the social conditions that will allow expert users to build and adapt the artifacts to be used by end users. In respond to this need, a special user role is specified: maieuta-designers. A maieuta-designer creates the necessary preconditions for facilitating expert users appropriate the design culture and technical notions necessary for the meta-task of artifact development and involving as many end users as possible in the process of continuous refinement of the artifact, by improving participation. The use of the term “maieuta” directly references the Socratic method of getting people acquire notions, motivations and self-confidence to undertake challenging tasks.

End users, expert users, meta-designers and maieuta-designers engage in certain interactions with each other as well as with the digital artifacts and tools causing the emergence of a co-evolution phenomenon. Meta-designers focus on designing and providing the most effective tools that may sustain the co-evolution between end users and expert users. Maieuta-designers facilitate the transition from the role of end user to the role of expert user thus empowering people to appropriate and contribute to their digital artifacts. If certain end users are not interested or fail to move towards the role of expert user, maieuta-designers may facilitate system evolution by systematizing the reporting of opportunities or shortcomings, as identified by end users, and proposing solutions handled by expert users or even suggest further technological contributions from meta-designers.

All the concepts presented above constitute the building blocks of the PerFECt framework as depicted in the figure below.
The interplay among the four roles of the framework give rise to two co-evolution processes as depicted in Fig. 1: The first one refers to the use of software devoted to the end user where there is continuous (cyclical) interaction between the end user and the system. This is depicted in Fig. 1 (left) with three homocentric cycles of arrows that represent the action-interpretation cycle at the lower level, the task-object cycle at the middle level and community-technology cycle at the upper level. In an analogous way, there is a second cyclical process depicted in Fig. 1 (right) that refers to the use of software devoted to expert user as building blocks of the system in continuous evolution. This process corresponds to yet another set of three homocentric cycles of the same nature: action-interpretation, task-object, and community-technology layers.

The inner interaction cycle in each co-evolution process refers to actions (triggered by the corresponding user or software) that are interpreted by the other party (software or user respectively). The task-object cycle in the middle refers to the co-evolution of the user task and the corresponding artifact within the boundaries of the System. Finally, an outer community-technology cycle captures the idea that the overall environment within which a user is working (community), co-evolves with the technology that supports the operation of this environment.

The PerFECt framework employs the notion of universality to addresses the issue of causality in digital representations, as Brenda Laurel puts it in her seminal book “Computers as Theatre”:

“The fact that people seek to understand causality in representational worlds provides the basis for Aristotle’s definition of universality. In the colloquial
an action is universal if everybody can understand it, regardless of cultural and other differences among individuals. This would seem to limit the set of universal actions to things that everyone on the planet does: eat, sleep, love, etc. Aristotle posits that any action can be “universalized” simply by revealing its cause; that is, understanding the cause is sufficient for understanding the action, even if it is something alien to one’s culture, background, or personal ‘reality’” (Laurel, 2013, p. 94).

Consequently, within the PerFEcT framework, the meta-task of expert users is to enable a universalization of physical objects by exploiting the available tools in the form of performative artifacts (pArtifacts) to account for the incorporation of the idea of performativity. The concept of performativity, as exposed by Cabitza & Simone (2015) emphasizes the fact that human behavior can be understood and analyzed by assuming that all human practices are performed so that actions can be seen as a public presentation of self. This is the conceptual basis of the methodological breakthrough titled the performative turn in cultural studies, social sciences, humanities and design. The term turn signifies the trend to reverse the ontological premises that reality corresponds to particular objects, entities, and configurations that exist in and of themselves exhibiting certain essential qualities towards a new central hypothesis that objects are textures of partially coherent and partially coordinated performances existing through multiple situated practices. Meaning making is essentially a social process. Knowledge is created through the actions of the members of a social structure. In this respect, there is a shift towards “the active social construction of reality rather than its representation” (Dirksmeier & Helbrecht, 2008, p. 4).

The roots of this approach can be attributed to the need to move beyond the prevailing focus on texts or symbolic representations to capture meaning. Performance is, above all, a meaning making bodily practice. Consequently, it is related to rituals and other forms of spectacles and social practices (Moumoutzis et al., 2017b). Beyond the main premises and the theoretical justification of the validity of performativity, one could attribute the significance of this paradigm to an inherent dramatic quality of human experience.

Beeman (1993) offers a very interesting comparison and in-depth analysis of the relation between theatre and other performative genres: Revolutions, public demonstrations, campaigns, strikes, and other forms of participatory public action all have performative dimensions sharing certain features with the fundamental ritual processes. Such social dramas involve a separation with normal structures of ongoing life, the entrance of groups of individuals into state of transition, and the re-integration of the individuals into a reconstructed social reality. Beeman (1993) goes on to analyze the interrelationship of stage drama, as a generalization of theatre, and social drama, as an inclusive term to describe all performative genres that aim at changing actual reality, employing a scheme initially proposed by Turner (1990). This scheme is depicted below:
Fig. 2. The interrelationship between social drama and stage or aesthetic drama. Concepts depicted following the ideas of Turner (1990).

Above the horizontal line Fig. 2 represents what is actual, visible and public while below the horizontal line what is hidden and virtual, i.e. implicit and internal. On the left of the vertical line social drama is represented, i.e. all performative genres related to social life while on the right any genre of cultural performance (aesthetic or stage drama). The arrows represent a circular process with a continuous feedback loop with four directions:

1. Manifest social drama (i.e. visible social and political action) feeds into the hidden space of aesthetic drama influencing both form and content of the latter.
2. The latent space of stage drama feeds into manifest performance. This way, stage drama operates as an active or “magic” mirror meant to do more than entertain being a meta-commentary on the major social dramas within the wider sociocultural context such as wars, revolutions, scandals, institutional changes etc.
3. Stage performance, within its own turn, feeds into the latent realm of social drama with its message and its rhetoric and partly account for its ritualization.
4. Finally, life itself stands as a mirror of art, of the stage drama, and the living perform their lives in a way that the protagonists of life are equipped with salient opinions, imageries and ideological perspectives created in stage drama.

The above feedback loop continues not as a cycle but rather as a helix: At each exchange new elements are added and other elements are left behind (forgotten or discarded). Beeman (1993) attributes human learning to experience and drama offers the deepest experience of all. By drama here, it is meant not only social drama, or stage drama alone, but both of them in an oscillatory process. Consequently, social dramas and aesthetic dramas in their mutual relationship and interaction produce the complex reality we all experience.

It is interesting to see how this conception of the reflective social process through which society looks at itself, learns from its experiences and continuously reconstructs or reinvents itself, resembles one of the most widely used models of learning: the learning cycle introduced by Kolb & Fry (1974) and further elaborated by Honey & Mumford (1982). This model distinguishes four phases in the learning process of an individual that proceed iteratively as depicted below:
In detail, the four learning phases along with their drama counterparts proceed as follows:

1. The process starts from *experiencing reality*, an activity that is preferable by *activist* learners that try to actually do things and have concrete experiences. This is analogous to overt social drama discussed already.
2. The next learning phase is *reviewing and reflecting on the concrete experience*, the preferred mode of learning for *reflectors* that observe (their own or other peoples’) actions. This is analogous to the latent realm of stage drama where social experiences are elaborated and give rise to art manifestations.
3. The third phase is *concluding from the experience* providing the means that will subsequently orient the individual in life. This is the preferred mode for *theorists*, i.e. people that build explanatory frameworks trying to find casual relationships and links to previous established norms and concepts in a way that resembles what is happening during the preparation and staging of drama manifestations.
4. Finally, the last phase is to *plan the next step* that will feed a new iteration. This is the preferred learning mode for *pragmatists* that try to exploit the knowledge accumulated in order to act in real life in an informed and purposeful manner. This is related to the latent realm of social drama where the art-refined social experience gets back into the social stage to enrich it with new concepts, plans and intentions.

The theatrical game presented in the next section, relies on this close connection of learning and drama presented above. In particular, it provides a creative environment where children can participate as actors or as members of an audience, in a spiral process, and have moments of experiencing phenomena pertaining to the binary system and how binary representations of numbers interact with each other, with moments of reflection, abstraction and, subsequently, planning for further action on the basis of the theatrical game that is at the center of the learning process.

The theatrical game design captures and uses the main components of the PerFECT framework. When children are acting they are essentially end users (in the terminology of the framework) that just need to follow certain rules with consistency. When children are part of the audience guided by a facilitator (this is the maieuta-designer in
the PerFECT framework) to explore and gradually develop their skills in manipulating the binary representation of numbers so that they could make arithmetic calculations, they are essentially in transition to become expert users, as the PerFECT framework suggests.

An very interesting aspect on how the Human Calculator theatrical game interprets the PerFECT framework and sheds new light on its applicability in designing collaborative learning experiences, is related to the idea of de-design (Cabitza, 2014). De-design evokes the idea that omitting and leaving out features from a design is just as critical to the success of a system as it is including them positively. This is connected to the fact that any feature does both afford and constrain interactions with and through the artifact, what is left out of it has the potential to be even more important than what designers put in it on purpose. This is a disciplined inaction that is intentional and goes beyond mainstream design approaches by offering opportunities for different interpretations of the information that need to be considered in user practices, and recognize the creative power of ambiguity. The relevance of de-design to learning and creativity, is thus evident.

Following a de-design approach, the Human Calculator theatrical game, takes the idea of universality, as used in the PerFECT framework, along with the underlying concept of causality, and uses it beyond digital technologies to account for a human body (or a constellation of human bodies) that behaves under certain rules. This way, the game uses the human body to enact the operation of circuits implementing a binary calculator. Extensions of the basic theme of the game, can give rise to alternative rules so that the participating children could creatively explore new situations and representations. In summary, the Human Calculator theatrical game, generalizes the notion of universal objects and universalizing assemblies (Fig. 1) to account for any kind of object that can follow well-known rules and corresponding constellations of such objects.

3 The Human Calculator Theatrical Game

The Human Calculator theatrical game addresses the need to introduce children to the binary system as the basis of modern digital computers. It offers an alternative learning path beyond mainstream approaches that heavily rely on a math-based presentation of the binary system and possibly use electrical/electronic circuits as learning tools. The game not only allows participants to understand how to convert numbers in binary, but also enables them to explore strategies (i.e. algorithms) to perform arithmetic calculations.

The Human Calculator theatrical game relies on the idea of theatrical workshops as a learning environment. In particular, it builds on the ideas of the Mathemart methodology and resembles the drama-based learning activities proposed by this methodology. Mathemart was created by Maurizio Bertolini in 2011 and employs the Social Community Theatre (SCT) methodology (Rossi Ghiglione, 2013). It addresses ways to bypass the fear of Mathematics and offers an engaging and playful way to deal with mathematical concepts of arithmetic, geometry and algebra. According to the details
that can be found at http://www.socialcommunitytheatre.com/en/projects/mathemart/
"the SCT methodology is used to get immersed in the game of mathematics by means of an overall approach involving mind and body, inborn creativity and involvement. The theatrical setting conveys a creative, playful and trusting atmosphere enabling students to freely explore without judging what they are doing. It encourages learning from mistakes in a sequence of trial and error. If a student is scared of mathematics, he cannot allow himself to make mistakes. His fear freezes his brain and logic skills and he cannot think straight. But it doesn’t mean that he is not really able to. A good theatrical setting can help students to forget that fear and to enjoy the possibility of learning by playing. In fact, within Mathemart training we do not talk about mathematics, we experience the subject by playing with mathematical relations and rules. Only after experiencing a concept we will formalize it.”

Following this philosophy, the Human Calculator theatrical game is based on role playing and subsequent reflection on the phenomena that emerge. Very simple rules are given to the participants to create an n-bit binary calculator using their bodies. To start the game, the facilitator splits the participants into two groups. One group (let us call it, the actors) enacts the Human Calculator while the other group (let us call it the audience) observes the operation of the Human Calculator explores its properties and reflects on the phenomena observed. The critical support of the facilitator is a prerequisite for the successful implementation of the activity. To enable the active participation of all, the participants alternate between the group of the actors and the audience.

After deciding on the group of the actors, the facilitator organizes them in a row, one behind the other, and asks them to learn and follow a simple rule: “Stand up, raise your hand and touch the shoulder of the child in front of you. Whenever somebody touches you, lower your hand. If somebody touches you again, raise your hand and touch the shoulder of the child in front of you. Continue this way forever”.

If 8 participants, following this rule are arranged in a sequence from right to left, the binary representation of a number n from 0 up to 255 can be found by touching the rightmost child n times. At the end, the digits of the representation are found as follows: Each child with the hand raised and touching the next child, represents a binary digit of zero. Each child with no hand raised, represents a binary digit of one. Note here that if the leftmost child raises its hand, it cannot touch another child on the right. However, it stills represents a digit of zero in such a case. The figure below depicts these two states for a child participating in the Human Calculator.
Fig. 4. The Human Calculator theatrical game distinguishes between two states representing zero (hand raised to touch the next participant) and one (stand up without touching anybody).

Using only these two states and organizing the group of the actors in a row, it is possible to find the representation of a given number as well as to perform all arithmetic operations. This justifies the name: Human Calculator. To depict the operation of the Human Calculator in the following paragraphs and the next two sections, we will use screenshots of a simulator of the game. The simulator has been developed in the Scratch programming environment (Maloney et al., 2010) and is available at: https://scratch.mit.edu/projects/410832633/

The figure below depicts the Human Calculator initialized. Eight children are organized in a row and all of them are in state 0 with their left hand raised and touching the next child on the row.

Fig. 5. The Human Calculator theatrical game started: Eight children, one behind the other, are organized in zero states, i.e. touching the child in front of them. Note that the leftmost child does not touch anyone else. Its state is zero as well.

Starting from this initial configuration, a binary representation of a number n can be found by touching the rightmost child n times. Note that a child changes state if someone touches it, following the rule of the game. Apart from the previous child touching the next one (whenever it moves into state 0), the operator of the Human Calculator can also touch it to make it change state. In the simulator, the operator is essentially the user of the simulator and touching a child is simulated by clicking the corresponding graphical representation of the child with the mouse.
During the actual game, the role of the operator is initially undertaken by the facilitator. At any point, any child from the audience could also step in and undertake this role to interact with the Human Calculator.

Let us see now what happens if the operator touches the rightmost child. As soon as the child feels the touch, following the rule of the game, it changes state to 1, as depicted in the figure below. The Human Calculator now represents number one in binary (00000001).

![Fig. 6. The Human Calculator theatrical game initiated to represent number 1.](image)

If the operator touches the rightmost child again, the child will move from state 1 to state 0, thus touching the next child to the left. As soon as the second child feels this touch, following the rule of the game, will go to state 1. This is depicted in the next figure. The corresponding number represented is two (00000010). Two is also the number of times the operator touched the first child so far.

![Fig. 7. The Human Calculator theatrical game: Representing number 2.](image)

Let us see now what will happen if the operator touches the first child from the right, for a third time. The child will change state from 0 to 1, lowering its hand. The second child will feel no touch so it will remain in the same state (1). The result is depicted in the figure below. The new set up corresponds to the binary representation of number three (00000011).
Let us depict what happens in one step more, when the operator touches for the fourth time the rightmost child. The rightmost child changes state, after the operator touches it, and raises its hand to touch the next child. In a similar way, this action will result in the second child changing its state to 0, raising its hand as well and touching the third child that will change its state from 0 to 1. The final setup is depicted below:

Fig. 9. The Human Calculator theatrical game: Representing number 4.

The process can continue until the desired number of touches has been made. The figure below depicts the situation after 13 touches to the rightmost child.

Fig. 10. The Human Calculator theatrical game: Representing number 13.

At this point, the facilitator can invite the children from the audience to experiment with the Human Calculator operation by interacting with it and see the binary representation of the numbers they choose. Furthermore, exploiting their natural curiosity, the facilitator guides to children to explore what happens if they try to touch any child in the row apart from the rightmost child. The objective is to understand and explore the consequences of this strategy: Each child actor in the sequence can be linked to a certain numerical value that corresponds to a power of two. The first child from the
right corresponds to $1=2^0$. The next child corresponds to $2=2^1$. The next one to $4=2^2$, the next one to $4=2^3$, etc. Consequently, to set up the Human Calculator to the binary representation of a certain number $n$, the operator can directly touch the actors that represent the 1 bits in the binary representation of $n$, if this binary representation is already known. This is the basis of performing additions and multiplications, as explained in the next section.

4 Addition and Multiplication in the Human Calculator
Theatrical Game

After mastering the basics of binary representation and being able to directly use binary representations of small integers, the Human Calculator game can be further used to introduce the participants to the algorithms for performing addition and multiplication in the binary system. Note here that the algorithms are essentially the same with the usual algorithms taught in primary schools for the decimal system. The only difference is that their binary counterparts are far easier to perform and, by exploiting the interactivity of the Human Calculator theatrical game, they can be more easily enacted using the bodies of the participating children instead of doing the same algorithms with pen and paper.

Let us see, then, how addition can be performed. We will assume again that the Human Calculator has been initialized with eight children, thus allowing the representation of up to 8-bit numbers. Recall from the previous section, that if you start from the representation of number zero (Fig. 5) the binary representation of a certain number $n$ can be directly “loaded” by touching the children that correspond to the 1 bits of the number $n$. Each child actor in the row corresponds to a certain value of power of two, as already explained in the previous section. When a child is touched, the corresponding power of two is added to the previous contents of the Human Calculator. Consequently, to add a number $x$ to the current contents of the Human Calculator, the children that correspond to the 1 bits of the binary representation of number $x$ should be touched. The sequence of touching them does not matter.

In the figure below, we show such a case of adding 13 with 3. We present the complete picture of the simulator of the game as it is organized to facilitate the exploration of operations. In particular, the simulator presents three rows of children, the top row stands for the first operand (A), the bottom row stands for the second operand (B) and the middle row stands for the operation result (A?B) as it is gradually formed by the actions of the operator. The screenshot below depicts the set up after entering the two operands, 13 at the top row (A=13) and 3 at the bottom row (B=3). The middle row is not set yet.
Fig. 11. Perform the addition 13+3: Representing the two operands.

The first step to do the addition is to load the first operand in the middle row. This is accomplished by touching, one by one, the three children that correspond to the 1s in the binary representation of number $A=13$. This is shown in the next figure.

Fig. 12. Perform the addition 13+3: Copying the first operand ($A=13$) to the middle row of the simulator that represents the desired result ($A?B$).

Next, we need to add the values that correspond to the two 1s of the binary representation of 3. We will first put the leftmost 1, i.e. the bit that corresponds to the value of $2^1$. This is done by touching the second child from the right at the middle row.
Initially, this child is in state 0 (as shown in the figure above. After we touch, it, it goes into state 1 as it is shown in the next figure. The corresponding partial result in the middle row is \(13+2 = 15\).

**Fig. 13.** Perform the addition \(13+3\): Adding the second operand (B=3) to the middle row of the simulator that represents the desired result (A?B). To do so, the leftmost bit of the binary representation of 3 is added first, giving a partial result of 15.

Finally, we need to add the rightmost bit of operand B=3, i.e. the bit that corresponds to value \(1=2^0\). This is done by touching the rightmost child in the middle row. Following the rules of the game, all four children at the right of the middle row will move to state 0 and the fifth child will move from state 0 to state 1. The final result is depicted in the next figure. The result of the addition has been calculated: \(13+3 = 16\).
Fig. 14. Perform the addition 13+3: Final result after adding the leftmost 1 bit of the second operand (B=3) to the middle row of the simulator that represents the desired result (A?B).

The process described so far for addition can continue to add any sequence of numbers. Each time, we add to the partial result so far, the next number of the sequence. The facilitator can invite the children from the audience to experiment with different ways of copying the 1 bits of the operands to the partial result and justify why there is no difference on which bit is copied first.

After exploring addition, the facilitator can invite the participants to play with multiplication. The most straightforward way of making a multiplication in the Human Calculator is by starting with the multiplicand and adding it k times where k is the value of the multiplier. This is something that the participants in the game are encouraged to find and explore for themselves. Then, they are guided to find a more efficient way of doing this series of multiplications exploiting the binary representation of the multiplier.

In particular, the participants are first invited to explore how a given number can be doubled by shifting its bits one position to the left. To multiply a number by $4=2^2$, its binary representation needs to be shifted to the left by 2 positions. And to multiply by $2^n$ a shift to the left by n positions is necessary.

After understanding the rules of multiplying by powers of two, the general procedure for multiplication can be presented: Perform a series of additions of the multiplicand shifted appropriately so that each 1 bit of the multiplier is used. For example, to multiply 12 by 5 start by setting A?B to 12 (i.e. the multiplicand shifted by 0 positions to correspond to the rightmost 1 digit of the multiplier) as shown below:
Fig. 15. Perform the multiplication 12 × 5: The multiplicand is entered in the first row (A=12) and the multiplier in the third row (B=5). The middle rows, starts from 0 and is first loaded with number 12 shifted by 0 positions, corresponding the rightmost bit of B=5. Next, we add the bits of A=12 shifted by 2 positions (corresponding to the leftmost 1 digit of 5). This essentially adds the number 12 × 4 = 48 to the previous partial result (A?B=12) reaching the final result of 60 (i.e. A?B = 12 × 5 = 60) as depicted in the screenshot below:

Fig. 16. Perform the multiplication 12 × 5: In the final step, the multiplicand is added in the middle row shifted by two positions. The value in the middle row becomes 60, which is the desired result.
5 Subtraction and Division in the Human Calculator Theatrical Game

To implement subtraction, the rule followed by the children actors in the Human Calculator theatrical game is changed as follows: "Whenever somebody stops touching you, lower your hand (stop touching the child in front of you). If somebody stops touching you again, raise your hand to touch the child in front of you. Continue this way forever". To enact the subtraction rule, the Scratch simulator available at https://scratch.mit.edu/projects/410832633/ supports two operational modes that can be activated by pressing the spacebar: The addition mode, presented with green background (as shown in the screenshots of the previous section) and the subtraction mode, presented with orange background. In the live version of the game, the change of mode can be signified by changing the lighting of the room, using flags that are visible from all children actors or using oral instructions, whatever is more convenient and effective.

Let us see what happens if number $A=13$ is first loaded and then subtraction mode is activated. The situation after entering the subtraction mode is depicted in the screenshot below:

![Fig. 17. Entering subtraction mode with a row of children representing number 13. The subtraction mode is depicted with orange background in the Human Calculator theatrical game simulator.](image)

Let us see what happens if the rightmost child, representing the value of $2^1=1$ changes state. To do so, the operator has to touch and then stop touching the child. Following the subtraction rule, the child feels that somebody stops touching it and changes its state from 1 to state 0 by raising its hand to touch the next child. The next child does not change state (as it would have happened in addition mode) because it feels somebody to touch it. But the subtraction rule orders to change state only when you feel somebody stop touching you. The new situation is depicted below:
Let us see next what happens if the leftmost child changes state once more, following the command of the operator. This time, it changes state from 0 to 1, meaning that the second child from the right will feel the child behind it to stop touching it and consequently it will change state from 0 to 1. The third child, then, will also change state going from 1 to 0. The final situation is depicted below:

It is not hard to see that whenever the rightmost child changes state, the subsequent application of the subtraction rule to the children on the left, brings the row of the children in a situation that corresponds to the binary representation of the number $n-1$ where $n$ is the number that was previously represented. Following similar observations and arguments as in the case of addition, it can be easily seen that each child on the row represents a value of power of two and changing its state, corresponds to subtracting the corresponding value from the number represented on the row.

The following screenshot depicts how an actual subtraction is performed in the Human Calculator theatrical game simulator. As in the case of addition, operands A and B are initialized and the A?B row starts with the contents of the A operand. This is all done in addition mode. Then the subtraction mode is activated. This is depicted in the figure below where we want to perform the subtraction 13-3.
Fig. 20. Perform the subtraction 13-3. Initialize the operands and the middle row to 13.

Then, to perform the subtraction, the 1 bits that correspond to operand B=3 should be transferred to A?B, i.e. 1=2^0 and 2=2^1 should be subtracted from the binary representation of 13. Following the subtraction rule, when the rightmost bit of 13 changes from 1 to 0, nothing more happens to the other bits. When the second bit from the right is changed from 0 to 1, following the subtraction rules, the third bit is also changed from 1 to 0. The fourth bit remains unchanged. The final set up is shown in the next screenshot.

Fig. 21. Perform the subtraction 13-3. Subtract the values (powers of two) that correspond to 3 from 13 to find the final result A?B=13-3=10
After mastering subtraction, division can be explored as a series of subtractions. In its simplest form, division corresponds to repetitive subtractions starting from the dividend and subtracting the divisor until zero is reached or a remainder (a positive number less than the dividend) is found. The number of subtractions made is the quotient of the division. We can speed up the process by following the long division algorithm or even explore other division strategies as listed in the corresponding Wikipedia lemma (https://en.wikipedia.org/wiki/Division_algorithm). Further details on this process are left for a future presentation. Finally, the representation of negative numbers can be explored. Following the rules of subtraction, if the Human Calculator is initialized to zero, the two’s complement representation of negative number can be found.

6 Conclusions and Future Work

The work reported in this paper is informed by PerFECt, a performative framework supporting collaborative learning and creativity that was initially developed to capture design principles suitable for the development of open learning environments (Mylonakis et al., 2011) to effectively support collaborative learning (Stylianakis et al., 2014) emphasizing the need to link learning to effective social structures. Consequently, this framework addresses issues related to the effective use of digital technologies to establish and sustain learning communities. The framework is also applicable in the analysis of serious games (Márkus et al., 2018) cultural heritage systems (Dochev et al., 2019) as important means to foster engagement and creativity in learning.

However, its core principles, as discussed in this paper, can also be employed to implement unplugged collaborative learning activities. This is the case of the Human Calculator theatrical game to creatively explore the mathematical principles of the binary system as the basis of modern digital computers. This way, we depart from the usual conception of Computer Science Education that assumes the mandatory use of electronic computing devices including robots, physical computing devices etc. Although all these devices convey an important perception of special purpose programming languages that can help learners understand the importance of computer coding and its potential for cultivating creativity, they cannot be compared with the human body in terms of flexibility and intimacy. These qualities of the human body and their relation to creativity can be easily observed in children’s play, especially in small ages.

The Human Calculator theatrical game, focuses on the body of the children and puts in action its flexibility, plasticity, and creative power to promote a creative exploration of the logic of the binary system. The overarching assumption is that the human body itself is perhaps the perfect means to study and set in motion interesting representations, to play with them and to gain experiential, embodied learning experiences that will shape both conscious and unconscious experiences in the same way that myths and fairy tales do, especially for small children. As a useful analogy, we consider music education and the use of musical instruments. As it is not mandatory to
use musical instruments in order to develop children’s musical perception, our approach tries to put forth the idea that computer science education is not necessary to rely on certain equipment or devices. Actually, the ability of the human body to make sounds (orally or even sounds made by body percussion) can be an excellent basis for musical education. In a similar manner, the human body and its ability to coordinate actions and follow rules can be an excellent basis for developing insights and knowledge related to the mathematical foundations of digital computers. To this end, the use of the Mathemart methodology provides a sound ground for many creative explorations.

The Human Calculator theatrical game is well suited for Early Childhood Education within the context of approaches related to the so called STEAM education. STEAM stands for Science, Technology, Engineering, Arts and Mathematics. Within this overall framework, several types of equipment and educational toys are currently used in early childhood classrooms to introduce computing principles and creative use of computers. In many cases, the use of mobile devices, in school or at home, raises significant concerns by both parents and educators related to computer addiction and the need to make sensible use of computers especially for little children. Indeed, early childhood is a special and critical period for human development as it is related to the development of fine motor skills, socialization and interaction with the others, the discovery of one’s own body and identity. Getting familiar with the body and physical materials are self-evident conditions of any pedagogical approach in this phase of human development. Consequently, the adoption of playful activities that put the children’s bodies in motion is well aligned with the pedagogical objectives of early childhood education. Note, here, that this emphasis on body movement can also be beneficial for students of higher grades as well as for university students. Such embodied knowledge approaches open up opportunities for deeper personalized learning experiences (Arapi et al., 2007; Yoshinov & Kotseva, 2016; Yoshinov et al., 2016) possibly combined with flipped teaching of mathematics (Lameras & Moutouzis, 2015).

There is a close relationship of the topics addressed by the Human Calculator theatrical game and official curricula of lower Secondary Education in Greece exploiting the official school curricula in Greece. In particular, the curricula of Mathematics in grade K-7 include positional numbers systems with an explicit reference to the binary system. Consequently, the Human Calculator game could be offered as an alternative creative approach to introduce the binary system, beyond current teacher-centered didactics. Furthermore, it can be used within the context of the Computer Science course in grade K-7 to K-9, where official Greek curricula include the binary system and the use binary coding to represent alphabetical character and strings. Within this topic, the Human Calculator theatrical game can be adapted to link binary representations to alphanumeric strings, and use the arithmetic operations to showcase algorithms related to cryptography.

Finally, future work will employ an alternative representation of the binary system using a binary counting table or abacus. The idea is to use this alternative representation to help children justify why the Human Calculator correctly computes the results of the arithmetic operations and better understand the applicability of the underlying
algorithms to other number systems, including the decimal system. The use of a binary abacus offers a personalized tool to explore the mechanics of the Human Calculator game when no other children are present to perform as the actors of the game (e.g. when at home).

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